at $\tilde{x}_D \leqslant 0.1$ and

$$\mathrm{Nu}_{0D} = B\exp(-m\tilde{x}_D) \tag{7}$$

at $\tilde{x}_D \geqslant 0.1$.

In contrast to the case of a symmetrically heated channel, the quantities A, B, m, and n depend on two temperature factors: $\psi_S = T_S/T_0$ and $\psi_W = T_W/T_0$. However, since the temperature factor $\psi_W$ is usually small in reactors with cooled walls, its effect can be ignored. In fact, at $T_0$ = 293 K, a change in $T_W$ from 323 to 573 K causes $\mathrm{Nu}_{0D}$ to change by 4-6% (curves 1 and 5 in Fig. 4).

We therefore determine these quantities as a function only of $\psi_S$:

$$A = 0.66\exp(0.17\psi_s),\ n = 0.39\exp(-0.053\psi_s),\ B = 1.3\cdot 0.1^{-12}A,$$
$$m = 0.45\psi_s + 1.46. \tag{8}$$

In the range $1 \leqslant \psi_S \leqslant 4.5$ the results of the numerical solution and results calculated from approximate formulas (6-8) differ by no more than 5% throughout the range $0.002 \leqslant \tilde{x}_D \leqslant 0.2$ except for the neighborhood of the point $\tilde{x}_D \approx 0.1$. Here, this difference increases to about 8%.

The effect of thermodiffusion and free convection can be evaluated from the relations shown in Figs. 2-4 and from Eqs. (2-5).

A similar calculation of heat fluxes in a reactor was performed in [4], while temperature and concentration profiles described by polynomials were presented in [3, 4].

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# EFFECT OF THE FORM OF THE TEMPERATURE DEPENDENCE OF SURFACE TENSION ON MOTION AND HEAT TRANSFER IN A LAYER OF LIQUID DURING LOCAL HEATING

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The nonsteady distribution of velocity and temperature in a layer of liquid during thermoconvective motion caused by local heating is calculated. The cases of an increasing and decreasing temperature dependence of surface tension are examined.

A large number of studies has been devoted to explaining the relative role of thermocapillary (TC) and thermogravitational (TG) convection in the motion of a liquid in a system with temperature gradients. These studies have become particularly important in connection with investigation of the behavior of liquids under conditions of reduced gravitation [1, 2].

Until recently, it was usually assumed when theoretically describing thermocapillary motion that surface tension decreased linearly with an increase in temperature (positive

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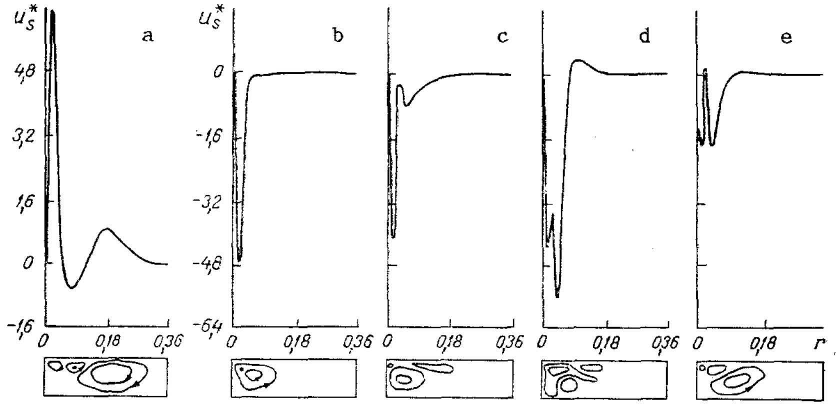

Fig. 1. Distribution of liquid velocity $u_s = u_s^* \cdot 10^3$ on on the surface of the layer along the radius for the cases examined: a) Ma = $9 \cdot 10^4$, Gr = 0, t = $12 \cdot 10^{-5}$; b) Ma = $-9 \cdot 10^4$, Gr = 0, t = $12 \cdot 10^{-5}$; c, d, and e correspond to simultaneous action of TC and TG convection, Ma = $-9 \cdot 10^4$, Gr = $10^7$, but different moments of time: c) $2 \cdot 10^{-5}$, d) $4 \cdot 10^{-5}$, e) $12 \cdot 10^{-5}$.

Marangoni effect) [3]. At the same time, it is known that a decreasing temperature dependence of surface tension is not universal. For certain aqueous solutions of alcohol, binary alloys, and liquid crystals, surface tension may have an extremum at a certain temperature and may both decrease and increase with an increase in temperature in different temperature intervals [4].

A change in the sign of the thermocapillary effect should lead to a substantial change in the character of the relationship between the thermogravitational and thermocapillary motion of liquids, since the directions of circulatory motion of the liquid generated by each of these effects become opposed to one another.

We will study motion and heat transfer in a liquid layer caused by local heating both in the presence and in the absence of thermogravitational convection in the case when there is a change in the direction of the thermocapillary forces.

We will examine the nonsteady development of convection inside a layer of thickness H of a liquid which is viscous and incompressible. The layer fills a plane cylindrical cuvette of radius R. At the initial moment of time, a region with elevated temperature and having the radius a' is created on the axis of the cuvette. It is assumed that temperature is distributed uniformly with respect to both height and radius in this region. The heat source is inactive at subsequent moments of time.

The mathematical formulation of the problem includes the axisymmetric nonsteady Navier—Stokes equation written in cylindrical coordinates in the Boussinesq approximation [5] and the heat conduction equation. Changing over to dimensionless variables and having selected the curl $\omega$, stream function $\psi$, and temperature $\Theta$ as the unknown functions, we write the initial equations in the form:

$$\frac{\partial\omega}{\partial t} + u\frac{\partial\omega}{\partial r} + v\frac{\partial\omega}{\partial z} + \frac{\omega u}{r} = \Pr\left(\nabla^2\omega - \frac{\omega}{r^2}\right) + \mathrm{Gr}\Pr^2\frac{\partial\Theta}{\partial r}, \tag{1}$$

$$\frac{\partial\Theta}{\partial t} + u\frac{\partial\Theta}{\partial r} + v\frac{\partial\Theta}{\partial z} = \nabla^2\Theta, \tag{2}$$

$$\nabla^2\psi - \frac{2}{r}\frac{\partial\psi}{\partial r} = \omega r, \tag{3}$$

$$\omega = \frac{\partial u}{\partial z} - \frac{\partial v}{\partial r},$$

$$u = \frac{1}{r}\frac{\partial\psi}{\partial z},\ v = -\frac{1}{r}\frac{\partial\psi}{\partial r},$$

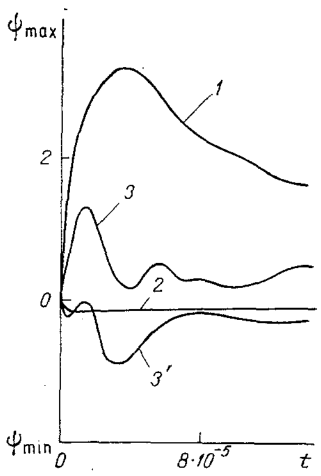

Fig. 2

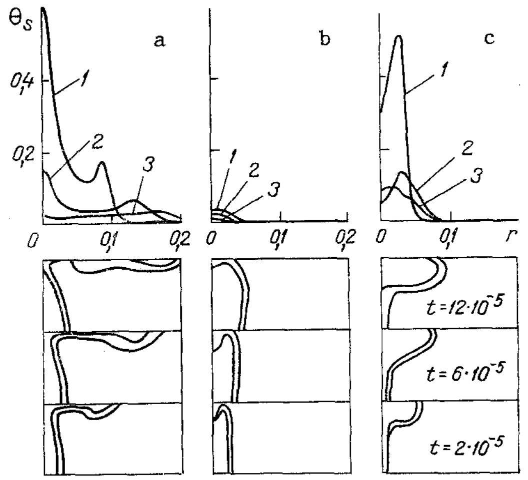


Fig. 3

Fig. 2. Dependence of the maximum $\psi_{max}$ and minimum $\psi_{min}$ values of the stream function on time: curve 1 corresponds to TC convection Ma > 0, Gr = 0; 2) anomalous TC convection Ma < 0, Gr = 0; 3 and 3') $\psi_{max}$ and $\psi_{min}$ with the simultaneous action of TC and TG convection Ma < 0, Gr ≠ 0.

Fig. 3. Temperature distribution on the surface of the layer in the radial direction: a) Ma > 0, Gr = 0; b) Ma < 0, Gr = 0; c) Ma < 0, Gr ≠ 0; 1-3) change in $\Theta_s$ at the times $2 \cdot 10^{-5}$, $6 \cdot 10^{-5}$, $12 \cdot 10^{-5}$; $\Theta$ = 0.1; 0.2.

$$\nabla^2 \equiv \frac{\partial^2}{\partial z^2} + \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}.$$

We assign conditions of balance of the viscous and TC forces (Marangoni effect) on the free surface (z = h) of the liquid

$$\frac{\partial u}{\partial z} = \mathrm{Ma}\frac{\partial \Theta}{\partial r}, \quad \mathrm{Ma} = -\frac{d\sigma}{dT}\frac{R(T_1 - T_0)}{\rho \nu \varkappa}. \tag{4}$$

Here, we will examine values of $d\sigma/dT$ which are both less than and greater than zero and, accordingly, positive and negative Marangoni numbers.

Conditions of adhesion of the liquid are assigned on the side walls (r = 1) and the bottom of the cuvette (z = 0)

$$\psi = 0, \quad \frac{\partial \psi}{\partial n} = 0. \tag{5}$$

The bottom, the lateral surface, and the free surface are considered to be thermally insulated

$$\partial \Theta / \partial n = 0. \tag{6}$$

Symmetry conditions are assigned on the cuvette axis

$$\psi(r = 0,\ z,\ t) = \omega(r = 0,\ z,\ t) = \frac{\partial \Theta(r = 0,\ z,\ t)}{\partial r} = 0. \tag{7}$$

The liquid is stationary at the initial moment of time

$$\psi(r,\ z,\ t = 0) = \omega(r,\ z,\ t = 0) = 0 \tag{8}$$

and the temperature distribution near the symmetry axis is given

$$\Theta(r,\ z,\ t = 0) = 1,\ 0 \leqslant r \leqslant a,\ \Theta(r,\ z,\ t = 0) = 0,\ r > a. \tag{9}$$

We used the following dimensionless variables in the formulas: $r = r'/R$, $z = z'/R$, $t = (\varkappa/R^2)t'$, $v = (R/\varkappa)v'$, $u = (R/\varkappa)u'$, $\omega = (\varkappa/R^2)\omega'$, $\psi = \psi'/(\varkappa R)$, $T' = T'(r, z, t)$ is temperature, $\Theta = (T' - T_0)/(T_1 - T_0)$, $h = H/R$, $a = a'/R$, $T_1$ is the temperature in the initial hot spot (9).

Problem (1-9) was approximated by means of an implicit difference scheme on a nonuniform grid [6]. In solving the curl equations (1), we used the boundary condition proposed in [7]. The steady-state Poisson equation for the stream function was solved iteratively on each time layer. The main results were obtained on a grid with 46 × 21 nodes. Since the largest temperature and velocity gradients occurred near the axis and the surface of the liquid filling the cylindrical vessel, the smallest radial steps were chosen near the axis and were equal to 0.002. The smallest steps in the height direction were chosen near the surface and were equal to 0.005. The time step was constant and was equal to $10^{-7}$. The characteristic time of the process in the dimensionless variables was $1.5 \cdot 10^{-4}$.

In studying TC convection for the typical case $d\sigma/dT < 0$, we used data for ethyl alcohol $Pr = 16$, $\varkappa = 9.23 \cdot 10^{-4}$ cm$^2$/sec, $Ma = 9 \cdot 10^4$, the dimensions of the cuvette corresponded to the dimensions in the experiment in [8]. In examining the case of an inverse dependence of surface tension on temperature, to reveal characteristic features of the phenomenon we took the same numerical values but the opposite sign for the Marangoni numbers. This corresponds to the actual values encountered, since $d\sigma/dT > 0$ [9] for certain aqueous alcohol solutions.

In the case of TC convection, a change in the sign of $d\sigma/dT$ changes the direction of vortical motion of the liquid. In the case of a positive Marangoni number with the simultaneous action of TC and TG convection, velocity increases due to the coincidence of the direction of circulation but the pattern seen does not change drastically in a qualitative sense. Of the greatest interest is the case of the simultaneous action of TC and TG convection with a negative Marangoni number, when the direction of motion generated by TC and TG forces is opposite, $d\sigma/dT > 0$, $Ma = -9 \cdot 10^4$, and $Gr = 10^7$.

To study the effect of the direction of the TC forces, we will examine three cases: 1) normal TC convection without allowance for gravitation, $d\sigma/dT < 0$, $Ma = 9 \cdot 10^4$, $Gr = 0$; 2) TC convection with an anomalous dependence of surface tension $\sigma$ on temperature and without allowance for gravitation, $d\sigma/dT > 0$, $Ma = -9 \cdot 10^4$, $Gr = 0$; 3) simultaneous action of TC and TG convection with a negative Marangoni number, $d\sigma/dT > 0$, $Ma = -9 \cdot 10^4$, $Gr = 10^7$.

The effect of the direction of surface tension on mass transfer in the liquid was followed from the results of calculation of the velocity of the liquid on the free surface. These results are shown in Fig. 1. Shown below the distributions are the streamlines in the cross section of the cuvette for the same moments of time as were used for velocity. Figure 1a corresponds to TC convection in the case of a positive Marangoni number and the moment of time $t = 12 \cdot 10^{-5}$, which is close to the completion of heat and mass transfer. Figure 1b shows TC convection with a negative Ma number and the same moment of time.

It is evident from a comparison of Fig. 1a and Fig. 1b that a change in the sign of $d\sigma/dT$ changes the direction of vortical motion of the liquid. Meanwhile, velocity is greater in the case $d\sigma/dT < 0$ (Fig. 1a). This is evidently connected with the fact that in the case $d\sigma/dT$, thermocapillary forces cause the liquid to flow from the cold region to the heated region, overcoming additional resistance. In the case $d\sigma/dT < 0$ (Fig. 1b), the liquid flows in the opposite direction.

Figure 1c-e shows the distribution of velocity on the surface and the streamlines for simultaneous TC and TG convection with a negative Marangoni number. The results are shown for the moments of time $2 \cdot 10^{-5}$, $4 \cdot 10^{-5}$, and $12 \cdot 10^{-5}$, respectively. The streamlines show that a complex flow structure develops inside the layer in this case as a result of the interaction of several toroidal vortices of different intensities. The velocity profile near the axis also has a complex structure, with two local maxima and minima. Over time, motion stabilizes with the formation of a slightly-varying profile having approximately the same local minima. It is evident that in the case of the combined action of TC and TG convection near the surface, motion generated by TC forces remains predominant in terms of velocity.

In all of the variants examined above, motion of the liquid is localized in a small region adjacent to the hot spot. It can be seen from the corresponding streamlines that this is related to the fact that vortical motion develops opposite the direction of initial motion after a certain amount of time has elapsed (the stream function takes negative values

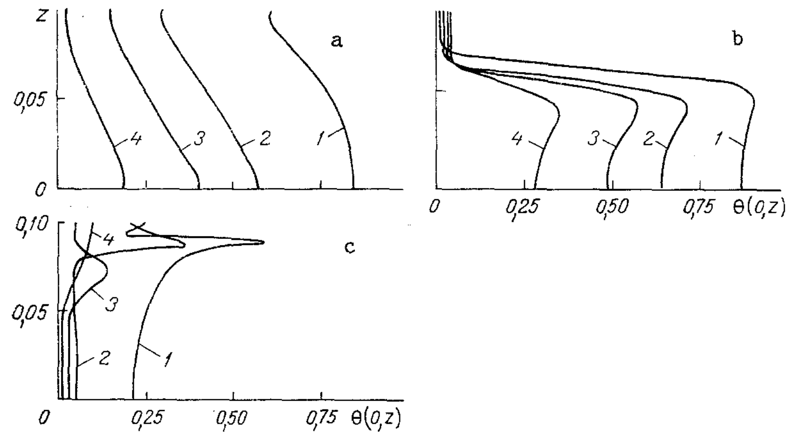


Fig. 4. Distribution of temperature on the axis Θ (r = 0, z, t) through the depth of the layer; curves 1, 2, 3, and 4 correspond to the moments of time $2\cdot10^{-5}$, $4\cdot10^{-5}$, $6\cdot10^{-5}$, $12\cdot10^{-5}$, respectively.

$\psi < 0$ and the direction of circulation of the liquid is counterclockwise). The development of this vortical motion prevents further movement of the liquid from the axis to the periphery.

Figure 2 shows the change in the rate of vortical motion over time for the above three cases. It should be noted that the value of $\psi_{max}$ characterizes the rate of circulatory motion of the liquid inside the layer in the clockwise direction, while $\psi_{min}$ denotes the same in the counterclockwise direction. A change in the sign of $d\sigma/dT$ in the absence of TG convection changes the direction of circulation in the toroidal vortex. With a positive Marangoni number ($d\sigma/dT < 0$), motion occurs from the center – curve 1 for $\psi_{max}$ ($\psi_{min}$ is not represented because it is close to zero). With a negative Marangoni number ($d\sigma/dT > 0$), liquid on the surface moves toward the axis of the cuvette and descends along the cylindrical hot spot. An abrupt and forced change in the direction of the particles near the surface leads to stagnation of the liquid. Thus, at Ma < 0, the velocity inside the layer (characterized by $\psi_{min}$ – curve 2) is considerably lower than with a positive Marangoni number.

In the case when TC and TG counteract one another, Ma < 0, Gr = $10^7$, a complex flow structure consisting of several interacting vortices of different intensities develops inside the layer (the isolines are shown in Fig. 1c-e). The relations $\psi_{max}(t)$ and $\psi_{min}(t)$ (curves 3 and 3' in Fig. 2) are oscillatory in character. Meanwhile, the maximum velocity in one of the toroidal vortices is reached when the velocity is minimal in another vortex.

The temperature distribution on the surface of the layer has a large effect on the character of motion near the surface. Figure 3 shows the temperature profiles for the cases a, b, and c. Shown below are the isotherms for the same moments of time. It follows from analysis of the isotherms that in the case of a positive Marangoni number $d\sigma/dT < 0$, Gr = 0 (Fig. 3a), heat is removed from the surface by convection and a second local heat source is formed near the surface. This situation leads to the development of reverse flows. Along with the initial temperature maximum located in the center, the surface-temperature profile shows a local temperature maximum located the same distance from the axis as the heat source formed inside the layer.

In the case $d\sigma/dT > 0$, Gr = 0 (Fig. 3b), thermocapillary forces cause heat to be removed from the surface to the interior of the liquid near the axis. In this case, the temperature on the surface quickly decreases to zero. The radius of motion of the liquid is considerably smaller than in the previous case.

When we also consider the force of gravity Gr = $10^7$ together with the anomalous temperature dependence $d\sigma/dT > 0$, heat is moved by convection inside the layer a relative short distance from the surface over the radius (Fig. 3c) as a result of gravitation. Surface thermocapillary forces impede this transfer, and the temperature of the surface becomes less than the temperature inside the liquid a short distance from the surface.

Figure 4 shows the temperature distribution on the axis of the vessel $\Theta$ (r = 0, z, t) through the depth of the layer for cases a, b, and c. An identical temperature $\Theta = 1$ is assigned on the axis at the initial moment of time. The subsequent change in temperature on the axis over time shows the process of heat removal from the heated zone.

In the case of normal TC convection Ma > 0 (Fig. 4a), heat is removed from the hot region along the surface. Here, heat moves from the center to the periphery. Heat is removed more rapidly from the upper layers, so that the temperature on the axis increases smoothly toward the bottom.

In the case of anomalous TC convection $d\sigma/dT > 0$, Ma < 0 (Fig. 4b), heat is moved by thermocapillary forces into the interior of the liquid near the axis, and the maximum temperature is seen a certain distance from the surface $z = h_1 < h$. Temperature decreases slightly approaching the bottom.

In the case of TC convection $d\sigma/dT > 0$ and the presence of gravitation (Fig. 4c), gravitational and thermocapillary forces compete with one another. The maximum temperature on the axis occurs a certain distance from the surface ($z = h_2 < h$) but closer to it than in the previous case $h_2 > h_1$. In contrast to the cases a and b examined above, temperature decreases sharply toward the bottom under the influence of gravity.

It follows from the above description of the character of TC and TG motion that when Ma < 0 and Gr $\neq$ 0, the change in the maximum rate of vortical motion is oscillatory in character. The temperature distribution along the axis has a distinct maximum inside the layer.

## NOTATION

r, radial coordinate; z, axial coordinate; t, time; v, vertical velocity; u, radial velocity; $\omega$, curl; $\psi$, stream function; a, radius of heated region; T, dimensional temperature; $\Theta$, dimensionless temperature; H, thickness of layer; h, dimensionless thickness of layer; R, radius of cuvette; $T_0$, initial temperature of medium; $T_1$, temperature of heated region; $\nu$, $\varkappa$, $\beta$, kinematic viscosity, diffusivity, and coefficient of thermal expansion, respectively; $\sigma = \sigma(T)$, surface tension; $\Pr = \nu/\varkappa$, Prandtl number; $\mathrm{Ma} = -\frac{d\sigma}{dT}\frac{R(T_1 - T_0)}{\varkappa\rho\nu}$, Marangoni number; $\mathrm{Gr} = g\beta R^3 (T_1 - T_0)/\nu^2$, Grashof number; Ra = GrPr, Rayleigh number; g, acceleration due to gravity. Indices: s pertains to values of the variables on the surface of the layer; ' pertains to the corresponding dimensional variables.

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